Fast Hough Transform Algorithm

Cvetan Cheshkov

CERN
Contents

- Motivation

- So called “Counting Rows & Gaps” Hough Transform:
  - Description of the algorithm
  - Practical implementation for fast filling of the Hough space

- New Hough space variables

- Results:
  - Efficiency, fakes
  - Resolution
  - Time consumption

- Conclusions & Plans
Motivation

- The original Hough Transform method designed within the HLT framework shows poor time performance (5-10s per patch for central events; PIII 800MHz)

- The relatively large amount of fakes limits the performance of the consecutive cluster fitter

- On the other hand, Hough Transform still seems a very promising in overcoming the common problems at high multiplicity events

- Is it possible to improve the time and tracking performances of the Hough method?

- Another interesting question is: could a stand-alone Hough Transform survive in high multiplicity environment?
Improving HT

- Originally HT was done filling the digits charge
  - Large amount of fakes in a high occupancy case
- Sophisticated peak finder

- Studying the HT performance it turned out that the performance can be improved by counting:
  - digits charge
  - #digits
  - #clusters (TPC rows)
“Counting Rows & Gaps” HT

- Naturally the studies lead to the idea to count consecutive TPC rows
- Idea: Good track candidate must occupy consecutive TPC rows
- Now the algorithm counts as well #gaps between occupied rows
- Weight used to fill HT space:
  - #rows (#gaps<N)
Example

Central HIJING $dN/dy=3500$, 0.2T
New Peak Finder

- The improvement in the HT quality
  - simpler PF
- The new PF looks for neighbor bins with the same weight
- Track parameters are extracted by averaging the peak edge points both in and $1/R$ (track curvature) directions
  - track is guided by the cluster borders
Fast Hough Transform (I)

- Assuming ordered TPC digits (in time bins, pads, padrows)
  new algorithm offers big space for speeding up!
- HT is monotonic along the padrows
  do it only for the first and last (in pad index direction) digits
  which belong to a cluster and fill at once the corresponding ribbon in the HT space
- Stopping rule using already accumulated #gaps
Fast Hough Transform (II)

- By introducing the new HT and substituting sin and cos by LUTs
  
a factor of 10 in the speed

- Which are the sources:
  - LUTs - factor of 2
  - HT algorithm – factor of 5

- Additional factor of 2
  - #gaps stopping rule
  - Code improvement
Hough Space (I)

- Two main problems with the current definition of the HT space variables $k(=1/R)$ and
  - The variables are **strongly correlated** “Butterfly”-like shape of the peaks
    - complex peak finder
    - relatively high fake peak rate
  - **Non-linear** functions in the filling of HT space
    - Need in LUTs inside HT filling loop
    - “Time consuming” FP operations inside HT filling loop

- **Can we find better solution?**
The choice of the new HT variables was inspired by:

- Search for natural variables connected to the physical setup of the TPC (an example is the seeding in the offline reconstruction)
- Search for variables which lead to linear HT

Why not use the Conformal Map transformation variables!
Hough Space (III)

- Lets define two curves inside a TPC sector by
  \[ \frac{x}{R^2} = \text{constant} = \frac{1}{2} \]

- Each primary track is represented by two points on these curves

  One can parameterize the track using

  \[ 1 = \frac{y}{R^1} \quad \text{and} \quad 2 = \frac{y}{R^2} \]

  For each space-point along the track trajectory:

  \[ \frac{(y/R^2 - 1)/(x/R^2 - 1)}{(2 - 1)/(2 - 1)} = \]
One can use the variables 1 and 2 to define the Hough space.

In this way each space-point inside a TPC sector will be represented in the Hough space by a straight line:

$$1 = A + B \times 2$$

with $A = \frac{y}{R^2} \left( 1 - \frac{2}{x/R^2} \right)$ and $B = \frac{x/R^2 - 1}{x/R^2 - 2}$

Linear filling of the Hough space!

The parameters A and B could be calculated in advance and used via LUTs.
Hough Space (V)

- Which position of the defining curves 1, 2?
- Requirement I: we need as uncorrelated as possible Hough peaks
- Requirement II: we have to preserve the monotonic behavior of the HT

Let's consider a primary track which crosses all 159 rows inside one sector:

- \((x/R^2 - 1)\) should change sign in the middle of the sector half of the SPs with \(B>0\) and half with \(B<0\)
- \((x/R^2 - 2)>0\) A always increases with \(y/R^2\)

Conclusion:

1 - in the middle (close to padrow 80) and
2 - at the outer edge (close to padrow 159)
Example

Parameterized HIJING dN/dy=8000, 0.4T

- Uncorrelated Hough peaks
- Gain of 30-40% in the time consumption
Testing conditions (I)

- Limits on Hough space variables correspond to a track with minimum $Pt[GeV/c] = B_{field}[T]$ which crosses the middle of the TPC sector.

- Hough space binning is fixed to $76(1) \times 140(2) \times 100(\eta)$; the size is roughly equal to the pad size in the corresponding TPC rows.

- Hough Transform runs stand-alone (without a consecutive cluster fitter).
Testing conditions (II)

- The method requires **new definitions** in order to determine the performance:
  - No clusters associated
    - assign *only* 1 MC label to each track
  - High occupancy (and therefore overlapping clusters) in general
    - does not affect the track parameters, but cause **appearance of “ghosts”**
    - fakes tracks  ghost tracks
  - If more than 1 track with the same MC label, take *randomly* one as good and second as fake (or ghost)
- **Good tracks** – primaries from offline comparison macro
Efficiency and Fakes (I)

0.4T

Tracking efficiency vs pt

- dN/dy=8000
- dN/dy=4000
- dN/dy=2000

P_T [GeV]

- 0.5 1 1.5 2 2.5 3 3.5 4
Efficiency and Fakes (II)

0.5T

Tracking efficiency vs pt

HIJINGparam dN/dy=8000
Full HIJING dN/dy~8000
Efficiency and Fakes (III)

0.2T

Tracking efficiency vs pt

\[ \frac{dN}{dy} = 8000 \]
\[ \frac{dN}{dy} = 4000 \]
Relative Pt resolution for tracks with $\text{Pt}_{\text{min}} < \text{Pt} < 2\text{GeV/c}$ in %

<table>
<thead>
<tr>
<th>$dN/dy$</th>
<th>8000</th>
<th>4000</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5T</td>
<td>$(0.9\ 0.2) + (2.3\ 0.3)\text{Pt}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4T</td>
<td>$(1.3\ 0.3) + (2.4\ 0.3)\text{Pt}$</td>
<td>$(0.9\ 0.5) + (2.7\ 0.5)\text{Pt}$</td>
<td>$(0.4\ 0.5) + (3.3+0.6)\text{Pt}$</td>
</tr>
<tr>
<td>0.2T</td>
<td>$(2.2\ 0.5) + (4.5\ 0.5)\text{Pt}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Relative Pt resolution for tracks with Pt>2GeV/c in %

<table>
<thead>
<tr>
<th></th>
<th>dN/dy=8000</th>
<th>dN/dy=4000</th>
<th>dN/dy=2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5T</td>
<td>6.5 0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.4T</td>
<td>8.3 0.6</td>
<td>7.8 0.8</td>
<td>8.7 1.3</td>
</tr>
<tr>
<td>0.2T</td>
<td>16.5 1.5</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
## Eta resolution in $10^{-3}$

<table>
<thead>
<tr>
<th></th>
<th>dN/dy=8000</th>
<th>dN/dy=4000</th>
<th>dN/dy=2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5T</td>
<td>6.0 0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.4T</td>
<td>6.4 0.1</td>
<td>6.3 0.1</td>
<td>5.7 0.2</td>
</tr>
<tr>
<td>0.2T</td>
<td>8.6 0.1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
## Resolution in mrad

<table>
<thead>
<tr>
<th></th>
<th>$dN/dy=8000$</th>
<th>$dN/dy=4000$</th>
<th>$dN/dy=2000$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0.5T</strong></td>
<td>6.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>0.4T</strong></td>
<td>7.0</td>
<td>6.6</td>
<td>6.6</td>
</tr>
<tr>
<td><strong>0.2T</strong></td>
<td>9.2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Resolution for parameterized HIJING (dN/dy=8000,0.5T) event

\[ \chi^2 / \text{ndf} = 3.797 / 5 \]
\[ p_0 = 0.9028 \pm 0.231 \]
\[ p_1 = 2.269 \pm 0.2599 \]

Relative Pt resolution (Pt>2 GeV)

<table>
<thead>
<tr>
<th>Entries</th>
<th>Mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>146</td>
<td>0.4618</td>
<td>7.463</td>
</tr>
<tr>
<td>\chi^2 / \text{ndf}</td>
<td>6.038 / 4</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>84.71 \pm 9.43</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.6781 \pm 0.6388</td>
<td></td>
</tr>
<tr>
<td>Sigma</td>
<td>6.472 \pm 0.461</td>
<td></td>
</tr>
</tbody>
</table>

Eta resolution

<table>
<thead>
<tr>
<th>Entries</th>
<th>Mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3411</td>
<td>-0.000144</td>
<td>0.000111</td>
</tr>
<tr>
<td>\chi^2 / \text{ndf}</td>
<td>95.89 / 20</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1462 \pm 34.9</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0001298 \pm 0.0001046</td>
<td></td>
</tr>
<tr>
<td>Sigma</td>
<td>0.0006017 \pm 0.000098</td>
<td></td>
</tr>
</tbody>
</table>

Pai resolution

<table>
<thead>
<tr>
<th>Entries</th>
<th>Mean</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3411</td>
<td>0.0003732</td>
<td>0.01164</td>
</tr>
<tr>
<td>\chi^2 / \text{ndf}</td>
<td>210.4 / 26</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1292 \pm 31.7</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.0001964 \pm 0.0001156</td>
<td></td>
</tr>
<tr>
<td>Sigma</td>
<td>0.006491 \pm 0.000109</td>
<td></td>
</tr>
</tbody>
</table>
Benchmarks

- HT was tested mainly on Itanium II (1.5 GHz, 6Mb L3 cache) machines
- Compiled with Intel ecc with aggressive optimization

<table>
<thead>
<tr>
<th></th>
<th>dN/dy</th>
<th>8000</th>
<th>4000</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hough Trans</td>
<td>29.4s</td>
<td>(136ms/patch)</td>
<td>17.3s</td>
<td>(80ms/patch)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calc Weight</td>
<td>1.2s</td>
<td></td>
<td>1.2s</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Finder</td>
<td>1.8s</td>
<td></td>
<td>1.3s</td>
<td></td>
</tr>
</tbody>
</table>

Time consumption is:
- strictly proportional to the HT space size
- roughly proportional to the event multiplicity
Demo!

- Demo with parameterized HIJING event \((dN/dy=8000)\)
on one of the GDCs (oplapro machines) used in the Computing DC

- The event is stored as DATE formatted file and is processed by a stand-alone executable used for monitoring

- Oplapro – Itanium II; 1.5GHz; 6Mb L3 cache
Conclusions

- The Hough Transform method time performance has been improved between one and two orders of magnitude.

- It has been proven that the HT could work even as a stand-alone tracker.

- The high occupancy does not affect significantly the performance of the algorithm.
Plans

- The new HT as well as the standard HLT tracker will be run during the event reconstruction in the second step of the Physics DC’04

- Try it on real physics trigger algorithms

- Still some problems with the efficiency, fakes, track merging to be solved